

Modification of heavy quark energy loss due to shear flow in hot QCD plasma

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We present the derivation of heavy quark energy loss in a viscous QCD plasma using kinetic theory. Shear flow changes both boson and fermion distribution functions which eventually modify heavy quark energy loss. Due to presence of non-zero flow gradient in the medium all the bath particles here are out of equilibrium. In these types of plasmas we show that without plasma screening effects heavy quark energy loss suffers similar type of infrared divergence as one encounters in non-viscous plasma. The screening effects are incorporated consistently through Hard Thermal Loop resummation perturbation theory in the small-momentum-transfer region to obtain finite leading order result in η/s . We also quantify the importance of the result and demonstrate that shear flow has significant effect on the heavy quark energy loss.

Recent years have witnessed significant progress in understanding the properties of hot and/or dense matter produced at relativistic heavy ion collider (RHIC) at BNL and the Large Hadron Collider (LHC) at CERN. Current research in this area has now generally accepted the fact that matter produced in these collider experiments behaves like nearly ideal fluid. Ideal hydrodynamics successfully estimates the lower bound on shear viscosity (η) and entropy density (s) ratio ($\eta/s = 1/4\pi$) of the fluid produced in the above mentioned experiments [1].

Several other experimentally measured quantities such as elliptic flow (v_2) (as a function of p_T), particle type and impact parameter are also well described by ideal hydrodynamics. But there are some limitations to this success also. Ideal hydrodynamics fails to describe the trend of v_2 beyond $p_T \sim 2$ GeV. Above $p_T \sim 2$ GeV, v_2 does not rise as predicted by nonviscous hydrodynamics. It also fails to describe certain relative trends observed in the baryon and meson elliptic flows [2, 3]. Current studies have been attributed in explaining these issues which reveal that invoking non-ideal viscous hydrodynamics falling trend of $v_2(p_T)$ in the higher p_T region can naturally be explained. Incorporation of non-ideal effects into the theory eventually modifies stress-energy tensor ($T^{\mu\nu}$). Along with the ideal part it also receives viscous correction ($\delta T^{\mu\nu}$) [2]. The modification of stress energy tensor due to non-zero flow gradient of the medium in turn modifies particle distribution function. The latter will now have a viscous part along with the non-viscous one ($f_0 + \delta f$). The correction term δf involves both the shear and the bulk viscosity coefficients and can be determined by solving Boltzmann equation. We however restrict ourselves only to the shear part [2].

Viscous corrected energy momentum tensor and/or the distribution function modifies various experimental observables like particle spectra, Handbury Brown-Twiss radii, or elliptic flow[4–6]. Current studies on the modification of photon and dilepton spectra due to non-zero

η have explored the fact that in case of photon it leads to larger thermalization time. It has also been argued in [7, 8] that non-ideal effects increase net photon yield due to slowing down of hydrodynamic expansion. In case of dilepton the space-time integrated transverse momentum spectra shows a hardening where the magnitude of the correction increases with the increasing invariant mass. In [9] the authors argue that the thermal description is reliable for an invariant mass $< 2\tau_0 T_0^2/(\eta/s)$, where, τ_0 is the thermalization time and T_0 is the initial temperature. Recently we have studied the effect of the shear flow on the fermionic damping rate [10], where it has been shown that like ideal Quantum Chromodynamic (QCD) plasma the magnetic sector remains logarithmic infrared divergent even after the incorporation of plasma screening effects through Hard Thermal Loop (HTL) mechanism. An attempt has also been made to calculate the drag and diffusion coefficients in viscous plasma numerically without considering plasma screening effects into the calculation [11]. In the present work we present a consistent formalism of derivation of the leading order heavy quark energy loss in viscous QCD plasma with plasma screening effects into consideration. In the current work we restrict to the first order viscous correction upto $\mathcal{O}(\eta/s)$ which allows us to present closed form analytical results.

Since heavy quarks are good probe of QGP, several calculations have already been performed over the last decades to estimate the heavy quark energy loss ($-dE/dx$) in ideal plasma [12–22]. Calculation of $-dE/dx$ in non-viscous QCD plasma has been plagued with infrared divergences. To deal with the problem, the usual way is to introduce Braaten and Yuan's prescription where one separates the integration into two domains: one involving the exchange of hard photons (or gluons), i.e., the momentum transfer $q \sim T$ and the other involving soft photons (or gluons) when $q \sim eT(gT)$ ($e, g \ll 1$). In case of the hard sector, one uses bare propagator and introduces an arbitrary cutoff (q^*) parameter to regularize the integration [23]. For the latter, on the other hand, the hard thermal loop (HTL) corrected propagator is used. These two domains, upon addition, yield results independent of the intermediate scale [24, 25].

In all the previous works on the heavy quark energy

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loss the authors have considered the fact that the bath particles are in equilibrium. In this paper we present a consistent formalism to calculate the heavy quark energy loss with plasma screening effects where bath particles are affected by the longitudinal flow of the medium.

The problem of motion of a heavy quark in a QCD plasma looks familiar to that of a problem of test particle in plasma. The problem thus reduces to Brownian motion problem where quarks are executing random motion in plasma. To start with we appeal to the Boltzmann equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_p \cdot \nabla_{\mathbf{r}} + \mathbf{F} \cdot \nabla_{\mathbf{p}} \right) f_p = -C[f_p], \quad (1)$$

right hand side of the above equation represents the collision integral and $v_p = p/E_p$ is the velocity of the particle. In absence of external force and gradients of temperature, velocity or density on the injected parton, the above equation becomes,

$$\frac{\partial f_p}{\partial t} = -C[f_p]. \quad (2)$$

In the present paper we consider a high energy heavy quark of mass m_Q and momentum p propagating through a QCD medium and scatters off the quarks and gluons of the bath. The heavy quark has energy E_p and the mass of the light quarks in the bath $m_q \ll gT$. The injected heavy quark has a fluctuating part ($f_p = \delta f_p$) and all the bath particles are affected by the flow of the medium. The equilibrium part of the injected heavy quark vanishes since, $E_p \gg T$. In the present work we are interested only in the $2 \rightarrow 2$ ($P + K \rightarrow P' + K'$) processes. The explicit form of the collision integral then becomes,

$$\begin{aligned} C[f_p] = & \frac{1}{2E_p} \int \frac{d^3k}{(2\pi)^3 2k} \frac{d^3p'}{(2\pi)^3 2E_p'} \frac{d^3k'}{(2\pi)^3 2k'} \delta f_p f_k (1 \pm f_{k'}) \\ & \times (2\pi)^4 \delta^4(P + K - P' - K') \frac{1}{2} \sum_{spin} |\mathcal{M}|^2. \end{aligned} \quad (3)$$

Note the difference of the thermal phase space here with that of the light quarks in [17]. While writing the above equation for high energetic parton the possibility of back scattering has been excluded and the approximation $(1 \pm f_{E_p}^0) \simeq 1$ has also been incorporated in the thermal phase space since $E_p \gg T$.

The effect of flow of the medium is incorporated through the distribution function of the bath particles. We write viscous corrected distribution function as $f_i = f_i^0 + \delta f_i^\eta$ ($\delta f_i^\eta \ll f_i^0$), where, $i = k, k'$. δf_i^η is the first order correction to the thermal distribution function.

Now, the expression for the energy loss can be obtained from Eqs.(2) and (3) with the help of the relaxation time approximation. With this approximation for the injected particle one writes,

$$\frac{\partial \delta f_p}{\partial t} = -C[f_p] = -\delta f_p \Gamma(p). \quad (4)$$

$\Gamma(p)$ can be identified as the particle interaction rate. The energy loss ($-dE/dx$) of heavy quark can be obtained by averaging over the interaction rate times the energy transfer per scattering and dividing by the velocity of the injected particle,

$$\frac{dE}{dx} = \frac{1}{v_p} \int d\Gamma \omega. \quad (5)$$

With the help of the Eqs.(3), (4) and (5) the heavy quark energy loss can be expressed as follows,

$$\begin{aligned} -\frac{dE}{dx} \Big| (p) = & \frac{1}{2E_p v_p} \int \frac{d^3k}{(2\pi)^3 2k} \frac{d^3p'}{(2\pi)^3 2E_p'} \frac{d^3k'}{(2\pi)^3 2E_k'} \\ & \times (f_k + \delta f_k^\eta) (1 \pm f_{k'} \pm \delta f_{k'}^\eta) \omega (2\pi)^4 \delta^4(P + K - P' - K') \\ & \frac{1}{2} \sum_{spin} |\mathcal{M}|^2. \end{aligned} \quad (6)$$

Now, the form of δf_i^η mentioned above depends on the various ansatz [3, 26–28],

$$\delta f_i^\eta = \chi(k) \frac{f_k^0 (1 \pm f_k^0)}{T} \hat{k}_i \hat{k}_j \nabla_i u_j. \quad (7)$$

In principle $\chi(k)$ can be determined from various microscopic theories as discussed in [3]. In most of the hydrodynamic calculations it is assumed that $\delta f_k \propto k^2 f_k^0$ and the proportionality constant is independent of the particle type. This is known as quadratic ansatz [3]. For the present case we are interested in a boost invariant expansion without transverse flow. In this scenario one can incorporate the viscous correction to the distribution function in the following way [2, 8, 11],

$$\delta f_i^\eta(k) = f_i^0 (1 \pm f_i^0) \Phi_i(k), \quad (8)$$

where,

$$\Phi_i(k) = \frac{1}{2T^3 \tau} \frac{\eta}{s} \left(\frac{k^2}{3} - k_z^2 \right). \quad (9)$$

The viscous modification holds true only in the local rest frame of the fluid and it contains the first order correction in the expansion of shear part of the stress tensor. τ is the thermalization time of the quark-gluon plasma (QGP) and the flow is along z axis. From the above expression this is also evident that the non-equilibrium part of the distribution function becomes operative only when there is a momentum anisotropy in the system.

In a medium with non-zero flow gradient with the distribution functions mentioned in Eqs.(8) and (9) the expression for energy loss can be written as,

$$\begin{aligned} -\frac{dE}{dx} \Big| (p) = & \frac{A_q}{2E_p v_p} \int_{p', k, k'} \sum_{i=1,2} \alpha_i \omega (2\pi)^4 \\ & \delta^4(P + K - P' - K') \frac{1}{2} \sum_{spin} |\mathcal{M}|^2 = \frac{-dE}{dx} \Big|_0^0 + \frac{-dE}{dx} \Big|_0^\eta \end{aligned}$$

where, \int_k is shorthand for $\int d^3k/(2\pi)^3 2E_k$ and $A_q = 2n_f/3$ (n_f is the number of flavor). In the above equation α_i 's contain the information of the viscous modified phase-space factor. α_1 contains the equilibrium part of the distribution functions, this gives us the usual heavy quark energy loss $-\frac{dE}{dx}|^0$ mentioned in [12–14, 16–22] where all the bath particles are in thermal equilibrium. $-\frac{dE}{dx}|^\eta$ is the viscous corrected energy loss. The equilibrium part of the phase space has the following form [10],

$$\alpha_1 = f_k^0(1 \pm f_{k'}^0), \quad (11)$$

and α_2 involves terms due to the viscous modifications to the light quark distribution functions for the bath constituents [10],

$$\alpha_2 \simeq [\Phi_k f_k^0(1 \pm f_k^0) \pm \Phi_k f_{k'}^0 f_k^0 \pm \Phi_{k'} f_k^0 f_{k'}^0]. \quad (12)$$

The above expression has been arrived at by neglecting terms $\mathcal{O}((\eta/s)^2)$ and $\mathcal{O}(f_i^3)$.

Inserting the above mentioned viscous corrected phase-space factor in Eq.(10), one obtains,

$$-\frac{dE}{dx}\Big|^\eta(p) \simeq \frac{A_q}{2E_p v_p} \int_{k,p',k'} [\Phi_k f_k^0(1 \pm f_k^0) \pm \Phi_k f_{k'}^0 f_k^0 \pm \Phi_{k'} f_k^0 f_{k'}^0] (2\pi)^4 \delta^4(P + K - P' - K') \omega \frac{1}{2} \sum_{spin} |\mathcal{M}|^2 \quad (13)$$

Since, in case of quark-quark (Q-q) scatterings small angle collisions give dominant contribution, we write the phase-space factor with the following approximations,

$$f_{k'}^0 = f^0(k + \omega) \simeq f_k^0 + \omega f_k^{0'} \\ \phi_{k'} = \phi^0(k + \omega) \simeq \phi_k^0 + \omega \phi_k^{0'}. \quad (14)$$

With the above expansion we approximate the phase-space factor upto $\mathcal{O}(f_i^2)$ and exclude higher order terms in ω . Hence,

$$-\frac{dE}{dx}\Big|_{Qq}^\eta(p) \simeq \frac{A_q}{2E_p v_p} \int_{k,p',k'} [\Phi_k f_k^0(1 - 3f_k^0)] (2\pi)^4 \delta^4(P + K - P' - K') \omega \frac{1}{2} \sum_{spin} |\mathcal{M}|^2. \quad (15)$$

To proceed further we have to know the interaction. For t channel Q-q scattering process the matrix element is given by [21, 22],

$$\frac{1}{2} \sum_{spin} |\mathcal{M}|_{Qq}^2 \propto \frac{\tilde{s}^2}{t^2}, \quad (16)$$

where, $\tilde{s} = s - m_Q^2$, s and t are the usual Mandelstam variables.

Now, we consider the case of the hard gluon exchange where the medium effects on the propagator can be ignored. In this case one can see that $-\frac{dE}{dx}|_{Qq}^\eta$ is infrared divergent ($-\frac{dE}{dx}|_{Qq}^\eta(p) \propto \int dq/q$) like non-viscous medium.

The usual way to handle these divergences is to incorporate the effects of plasma screening. The method of calculating the effects of screening was developed by Braaten and Yuan [23]. As mentioned earlier this involves introduction of an arbitrary momentum scale q^* to distinguish the region of hard momentum transfer from the soft region. The contribution from the hard momentum region is calculated using tree-level scattering diagrams whereas HTL propagator is required for the soft momentum transfer. The matrix amplitude with HTL resummed gluon propagator is necessary to evaluate $-\frac{dE}{dx}|_{Qq}^\eta$ in the latter domain. In the large wavelength limit $q \ll T$ this reduces to,

$$\frac{1}{2} \sum_{spin} |\mathcal{M}|_{Qq}^2 = 32g^4 E_p^2 k^2 \left[\frac{1}{(q^2 + m_D^2)^2} + \frac{\left(v_p^2 - \frac{\omega^2}{q^2}\right) q^2 \cos^2 \phi}{\left(1 - \frac{\omega^2}{q^2}\right) \left(q^6 + \frac{\pi^2 \omega^2 m_D^4}{16}\right)} \right], \quad (17)$$

where, $m_D \sim gT$ is the Debye mass. Evaluating both the hard and the soft sectors and restricting ourselves mainly to the leading logarithmic contribution one obtains,

$$-\frac{dE}{dx}\Big|_{Qq,l}^\eta(p) \simeq \left(\frac{\eta}{s}\right) \mathcal{C}_1 \left[f_1(v_p) - 2f_1(v_p) \log \left| \frac{q_{max}}{m_D} \right| \right], \\ -\frac{dE}{dx}\Big|_{Qq,t}^\eta(p) \simeq \left(\frac{\eta}{s}\right) \mathcal{C}_2 \left[\frac{4}{15} f_2(v_p) + f_2(v_p) \log \left| \frac{2q_{max}}{\sqrt{\pi} v_p m_D} \right| \right], \quad (18)$$

where $-\frac{dE}{dx}|_{Qq,l}^\eta$ and $-\frac{dE}{dx}|_{Qq,t}^\eta$ denote the longitudinal and the transverse contributions to the energy loss ($-\frac{dE}{dx}|_{Qq}^\eta = -\frac{dE}{dx}|_{Qq,l}^\eta - \frac{dE}{dx}|_{Qq,t}^\eta$). The functions mentioned in the above equation have the following forms,

$$\mathcal{C}_1 = \frac{A_q g^4 T}{(2\pi)^3 \tau v_p^2} \left(-\frac{7\pi^4}{60} + \frac{27\text{Zeta}[3]}{2} \right), f_1(v_p) = \frac{v_p^5}{5} - \frac{v_p^3}{3}, \\ \mathcal{C}_2 = \frac{A_q g^4 T}{(2\pi)^3 2\tau v_p^2} \frac{4}{15} \left(-\frac{7\pi^4}{60} + \frac{27\text{Zeta}[3]}{2} \right), f_2(v_p) = v_p^5. \quad (19)$$

where, q_{max} in Eq.(18), can be approximated as $\sim \sqrt{E_p T}$ from the kinematics.

To obtain t channel contribution of quark-gluon (Q-g) scatterings from Eq.(18) the equation excluding A_q has to be multiplied with the color factor $A_g = (N_c^2 - 1)/2 = 4$ (N_c is the number of color). The total t channel contribution to the $-dE/dx$ is then given by $-\frac{dE}{dx}|_Q^\eta = -(4 + 2n_f/3) \frac{dE}{dx}|_{Qq}^\eta$. In this regard it would be important to recall the expression for the ideal heavy quark energy loss (in t channel) in [14, 16, 22].

We now present the derivation of the contribution of Q-g scatterings to the heavy quark energy loss in s and

u channels. One starts with the following expression,

$$-\frac{dE}{dx}\Big|_{Qg}^{\eta}(p) \simeq \frac{A_f g^4}{v_p} \int \frac{d^3 k d^3 k' d^3 p'}{(2\pi)^5 2E_p 2E_{p'} 2k 2k'} \\ \times [\Phi_k f_k^0 (1 + f_k^0) + \Phi_{k'} f_{k'}^0 f_k^0 + \Phi_k f_{k'}^0 f_k^0] \\ \times \delta^4(P + K - P' - K') \left[\frac{-\tilde{u}}{\tilde{s}} + \frac{-\tilde{s}}{\tilde{u}} \right] \quad (20)$$

where, $\tilde{u} = u - m_Q^2$ and $A_f = 16/9$. The k' integration can be expressed as

$$\int_{k'} \frac{\Phi_k f_k^0 (1 + f_k^0) + \Phi_{k'} f_{k'}^0 f_k^0 + \Phi_k f_{k'}^0 f_k^0}{2k'} \\ \times (2\pi)^4 \delta^{(4)}(P + K - P' - K') \\ = 2\pi \Phi_k f_k^0 (1 + f_k^0) + \Phi_{k+\omega} f_{k+\omega}^0 f_k^0 + \Phi_k f_{k+\omega}^0 f_k^0 \\ \Theta(k + \omega) \delta((K + Q)^2). \quad (21)$$

The integration over $\phi_{p'}$ can be done with the help of the delta function as shown below,

$$\int_0^{2\pi} d\phi \delta((K + Q)^2) = \frac{2}{\sqrt{f}} \Theta(f), \quad (22)$$

where, $f = B^2 - A^2$. A and B can be expressed in terms of the Mandelstam invariants [21, 22],

$$A = s - m_Q^2 + t - 2kE_{p'} + 2kp' \cos \theta_k \cos \theta_{p'}, \\ B = 2kp' \sin \theta_k \sin \theta_{p'}. \quad (23)$$

The variables can be changed from p' and $\cos \theta_{p'}$ to t and ω respectively by the following transformation,

$$t = 2(m_Q^2 - E_p E_{p'} + pp' \cos \theta), \\ \omega = E_p - E_{p'}. \quad (24)$$

With this, Eq.(20) now becomes,

$$-\frac{dE}{dx}\Big|_{Qg}^{\eta}(p) \simeq \frac{A_f g^4}{16\pi^2 v_p p E_p} \int_k \frac{1}{2k} \int_{-\infty}^0 dt \int_{-\infty}^{\infty} \frac{d\omega \omega}{\sqrt{f(\omega)}} \\ (\Phi_k f_k^0 (1 + f_k^0) + \Phi_{k+\omega} f_{k+\omega}^0 f_k^0 + \Phi_k f_{k+\omega}^0 f_k^0) g(s, t, \omega), \quad (25)$$

where, $g(s, t, \omega)$ depends on the Mandelstam variables and exchanged energy. Bounds on the integrals ω and t arise from the condition $f = B^2 - A^2 \geq 0$. $f(\omega)$ can now be written as follows $f(\omega) = -a^2 \omega^2 + b\omega + c$ [21, 22]. The coefficients of the above equation are [21, 22],

$$a = \frac{s - m_Q^2}{p}, \\ b = -\frac{2t}{p^2} (E_p (s - m_Q^2) - k(s + m_Q^2)), \\ c = -\frac{t}{p^2} [t((E_p + k)^2 - s) + 4p^2 k^2 - (s - m_Q^2 - 2E_p k)^2]. \quad (26)$$

$f(\omega)$ is positive only in the domain $\omega_{min} \ll \omega \ll \omega_{max}$, where the discriminant $D = 4a^2 c + b^2$ is positive. Thus ω_{min}^{max} and D can be evaluated along the line described in [21, 22]. The condition $D \geq 0$ leads to the $2 \rightarrow 2$ scattering processes with one massless and one massive particle in the limit $t_{min} \leq t \leq 0$ with $t_{min} = -(s - m_Q^2)^2/s$ [21, 22]. Like Q-q scatterings here also we neglect terms which are more than $\mathcal{O}(f_i^2)$ and higher order in ω .

Evaluation of the ω integral in Eq.(25) gives,

$$I_{\omega} = \int_{\omega_{min}}^{\omega_{max}} d\omega \frac{\omega}{\sqrt{f(\omega)}} = \text{Re} \int_{-\infty}^{\infty} d\omega \frac{\omega}{\sqrt{f(\omega)}} = \frac{\pi b}{2a^3}. \quad (27)$$

With the help of the above expression Eq.(25) reduces to,

$$-\frac{dE}{dx}\Big|_{Qg}^{\eta}(p) \simeq \left(\frac{\eta}{s}\right) \frac{A_f g^4}{4T^3 \tau \pi v_p} \int_k \left(\frac{1}{3} - \cos^2 \theta_{kz}\right) \\ (k^2 f_k^0 (1 + 3f_k^0)) \left(1 - \frac{(s + m_Q^2)k}{(s - m_Q^2)E_p}\right) \int_{t_{min}}^0 dt \\ \frac{-t}{(s - m_Q^2)^2} \left[\frac{-\tilde{u}}{\tilde{s}} + \frac{-\tilde{s}}{\tilde{u}}\right]. \quad (28)$$

As mentioned earlier we are interested in the energy loss of a high energetic parton $E_p \gg m_Q^2/T$, which implies $s = m_Q^2 + 2PK \sim \mathcal{O}(E_p T) \gg m_Q^2$. In this domain $((s + m_Q^2)k/(s - m_Q^2)E_p) \rightarrow 0$. Finally the expression for the Q-g scatterings in the s and the u channel reduces to,

$$-\frac{dE}{dx}\Big|_{Qg}^{\eta}(p) \simeq \left(\frac{\eta}{s}\right) C_3 \left[\frac{-11T^4}{18} \left(18\zeta(3) - \frac{2\pi^4}{15}\right) \right. \\ \left. - \frac{T^3 m_Q^2}{3E_p} \left((\pi^2 - 4\zeta(3)) \ln \left| \frac{4E_p T}{m_Q^2} \right| - .162225 \right) \right] \quad (29)$$

$C_3 = A_f g^4 / (32T^3 \tau \pi^3 v_p)$. The leading logarithmic term of Q-g scatterings (both in s and u channels) in non-viscous medium is given in [22]. The final expression of heavy quark energy energy loss can be obtained by adding Eqs.(18), and (29) along with the ideal contribu-

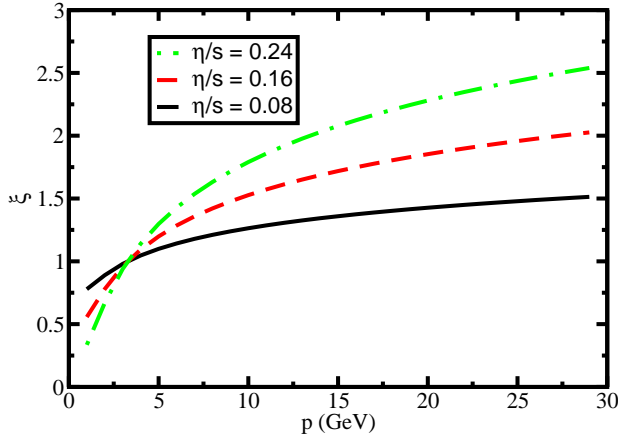


FIG. 1: (Color online) The variation of ξ of charm quark with momentum at $T = 0.225$ GeV, $\tau = 0.3$ fm, $m_D = 0.5$ GeV and $\alpha = 0.3$.

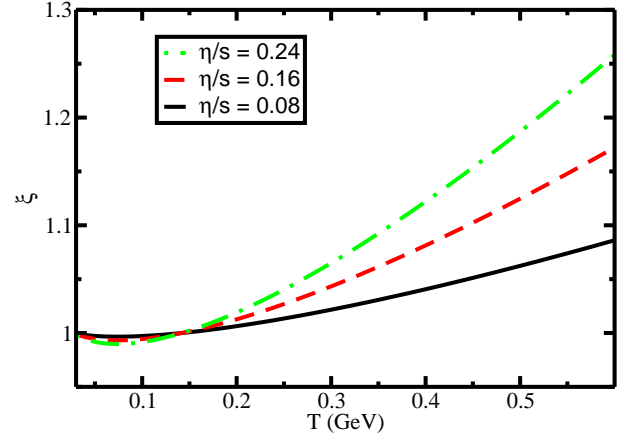


FIG. 2: (Color online) The variation of ξ of charm quark with temperature at $p = 5$ GeV, $\tau = 0.3$ fm, $m_D = 0.5$ GeV and $\alpha = 0.3$.

tions,

$$\begin{aligned}
-\frac{dE}{dx}\bigg|_p &= \frac{dE}{dx}\bigg|_{Qq}^0 + \frac{dE}{dx}\bigg|_{Qg}^0 + \frac{dE}{dx}\bigg|_{Qq}^\eta + \frac{dE}{dx}\bigg|_{Qg}^\eta \\
&= \frac{dE}{dx}\bigg|_{Qq}^0 + \frac{dE}{dx}\bigg|_{Qg}^0 + \left(\frac{\eta}{s}\right) \left(4 + \frac{2n_f}{3}\right) \left(-\frac{7\pi^4}{60}\right. \\
&\quad \left. + \frac{27\text{Zeta}[3]}{2}\right) \left[\frac{g^4 T}{(2\pi)^3 \tau v_p^2} \left\{f_1(v_p) - 2f_1(v_p) \log \left|\frac{q_{max}}{m_D}\right|\right\}\right. \\
&\quad \left. + \frac{g^4 T}{60\pi^3 \tau v_p^2} \left\{\frac{4}{15} f_2(v_p) + f_2(v_p) \log \left|\frac{2q_{max}}{\sqrt{\pi} v_p m_D}\right|\right\}\right. \\
&\quad \left. + \left(\frac{\eta}{s}\right) \frac{A_f g^4}{32T^3 \tau \pi^3 v_p} \left\{\frac{-11T^4}{18} \left(18\zeta(3) - \frac{2\pi^4}{15}\right)\right.\right. \\
&\quad \left.\left. - \frac{T^3 m_Q^2}{3E_p} \left((\pi^2 - 4\zeta(3)) \ln \left|\frac{4E_p T}{m_Q^2}\right| - .162225\right)\right\}\right]. \quad (30)
\end{aligned}$$

The above expression for energy loss in a plasma where all the bath particles are affected by the shear flow of the medium reveals the fact that similar to the heavy quark energy loss in a non-viscous medium this also suffers from infrared divergence for bare propagation. Using HTL propagator we obtain closed form leading order analytic result first order in viscous correction $\mathcal{O}(\eta/s)$.

To quantify the nature of heavy quark energy loss we here plot fractional energy loss ($\xi = \frac{dE}{dx} / \frac{dE}{dx}\bigg|_p^0$) with momentum (p) in Fig.(1) and with temperature (T) in Fig.(2) for charm quark. In Fig.(1) we observe the momentum variation of ξ for charm quark. From the plot it is clear that at $T = 0.225$ GeV and at low momentum $p < 5$ GeV increase in η/s decreases the relative energy loss, whereas in the high momentum region $p > 5$ GeV opposite nature can be seen. Fig.(2) depicts the variation of ξ with temperature at $p = 5$ GeV. In Fig.(2) note that at lower temperature regime ξ decreases and after $T = 0.15$ GeV it starts to increase. Hence,

there is a crossover of trend of ξ with η/s at $T \sim 0.15$ GeV. The nature of the two plots can be understood from the flow modified phase space factor. Since, the anisotropic phase-space part is negative in the low momentum/temperature region this gives negative contribution to the total energy loss whereas in the higher momentum/temperature region this contributes positively.

To summarize, in the present work we have calculated heavy quark energy loss in a medium where all the bath particles are affected by the shear flow of the medium. It has been shown in the text that shear viscosity enters into the calculation through the viscous corrected phase-space factor and this modifies the result of the energy loss. In the present work only elastic Q-q, Q-g scatterings have been considered and the leading order correction terms in η/s to the energy loss have been estimated. We observe that in case of viscous medium also bare gluon propagation gives rise to logarithmic infrared divergent $\int dq/q$ result similar to the case of non-viscous medium. HTL resummed propagator is used to circumvent the problem. While performing the calculation only small angle contributions are considered since these provide us the dominant contribution to the energy loss. Along with the above mentioned points it has also been considered that only longitudinal shear flow is present in the current derivation. The approximations mentioned above allow us to present leading logarithmic, first order correction term in η/s of the heavy quark energy loss. Moreover, one of the interesting findings of the present work is, the plasma effects in a viscous medium increases heavy quark energy loss in the high momentum region ($p > 5$ GeV). In the low temperature region ($T < 0.15$ GeV) the energy loss shows opposite trend in comparison to the high temperature one. Findings of the present work will have significant consequences in studying different observables like nuclear modification factor, particle spectra in recent heavy ion collision experiments.

I am indebted to Late Prof. A. K. Dutt-Mazumder who

introduced me to this topic and his fruitful discussions motivated me to initiate the present work. I would also like to acknowledge Prof. J. P. Blaizot, Prof. P. K. Roy

and Dr. S. K. Das for their valuable discussions and comments.

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